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Engineering Notes

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Simple Quantitative Method to Compare Aircraft Wing Mode Shapes

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Introduction

THE importance of mode shapes is undeniable in contemporary flutter analyses. Not only are modes used to transform the governing equations from spatial to modal coordinates, but they are also used to transform flutter solution eigenvectors back to physical space to facilitate viewing of the critical flutter mechanism components. Mode shapes have even been used to gain insight into nonlinear aeroelastic behavior, such as limit cycle oscillations [1]. Mode shapes are so commonplace and easy to generate that their ability to overcome weaknesses inherent in flutter solution methods is not appreciated. In essence, modes are powerful tools that are not consistently exploited to their full potential.

One cause for the underutilization of mode shapes could be the difficulty in quantitatively comparing them in an efficient manner with other modes. Quantitative comparison of modal frequencies is convenient, but a subjective assessment of the deflection mode shapes is the convention. Because of this subjectivity, the results of modal comparisons in this fashion could be regarded as less reliable or of limited utility. The situation is further impaired for complex or large models that are often cumbersome and sometimes obscure key shape features of interest.

Anyone who has evaluated large quantities of flutter analyses is aware of the benefits of being able to compare mode shapes of different dynamical systems. These mode shapes can be indicative of energy-extraction and damping pathways within the structural system. They can provide insight into the similarity of predicted solution results, thereby reducing the number of critical analysis cases that may require flutter flight testing. They can also aid in the identification of critical flutter modes during flight testing, which is necessary for damping determination [2] and flutter margin [3,4] methods.

Many methods are available to identify modes, but methods to explicitly compare one mode shape with another are not well documented in the literature. The methods that are available for mode shape comparisons are predominantly visual-based, such as 3-D still plots, animations, and node lines, although some work to extend these methods to quantitative approaches is available. Of particular interest to the flutter engineer is Maxwell's work (as reported in [5]) to depict the wingtip deformation characteristics of the *flutter* mode shape and Northington's [6] basis vector approach. There are also strain energy methods [7].

In short, it appears that many in-house methods for mode shape comparison likely exist, but they are only passed from generation to generation internally and are not disseminated widely to the engineering public. The development and dissemination of concise and distinctive mode shape description methodologies would allow the flutter engineering community to take better advantage of the inherent utility of mode shapes. Some applications that would become more practicable are sorting, tracking, and screening of modes for large quantities of analyses and using modes in alternative solution methodologies, such as artificial neural networks [8].

This Note describes a simple method in which curve-fit equations are used to depict the mode shapes of the wing. Some examples will be presented that compare the conventional 3-D still plots of the mode shape to the fitted curves as well as the coefficients of the curve-fit equations.

Methodology

The method presented here employs polynomial curve fits of the wing to reduce the number of quantitative descriptors necessary to define the spatial character of the mode shape. This approach was inspired by surface spline [9] techniques routinely employed in flutter analysis. For the work presented here, however, a functional representation of the wing deflections was desired, as opposed to merely interpolating or matching deflection values. In other words, a simple equation that concisely described the *general* character of the mode shape was sought, rather than a high-fidelity definition of the modal motion.

Because wing structural vibration modes are a primary area of interest to the flutter engineer, the method presented here confines itself to the main wing planform, although its use on other lifting surfaces is certainly applicable. This approximation is considered to be reasonable and practical because the wing modes are influenced by the structural dynamics and aerodynamics of any underwing attached structures and consequently possess the pertinent wing-flutter-critical effects. Deflections at the forward and aft spars are

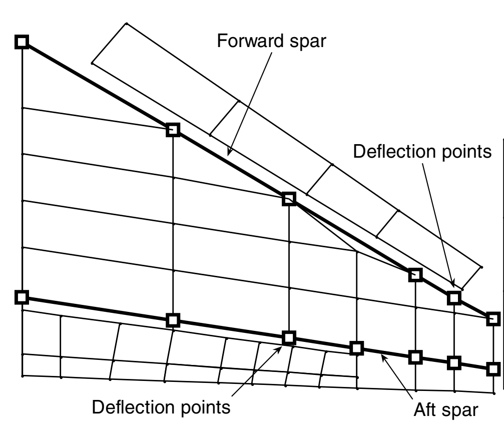

Fig. 1 Wing planform representation.

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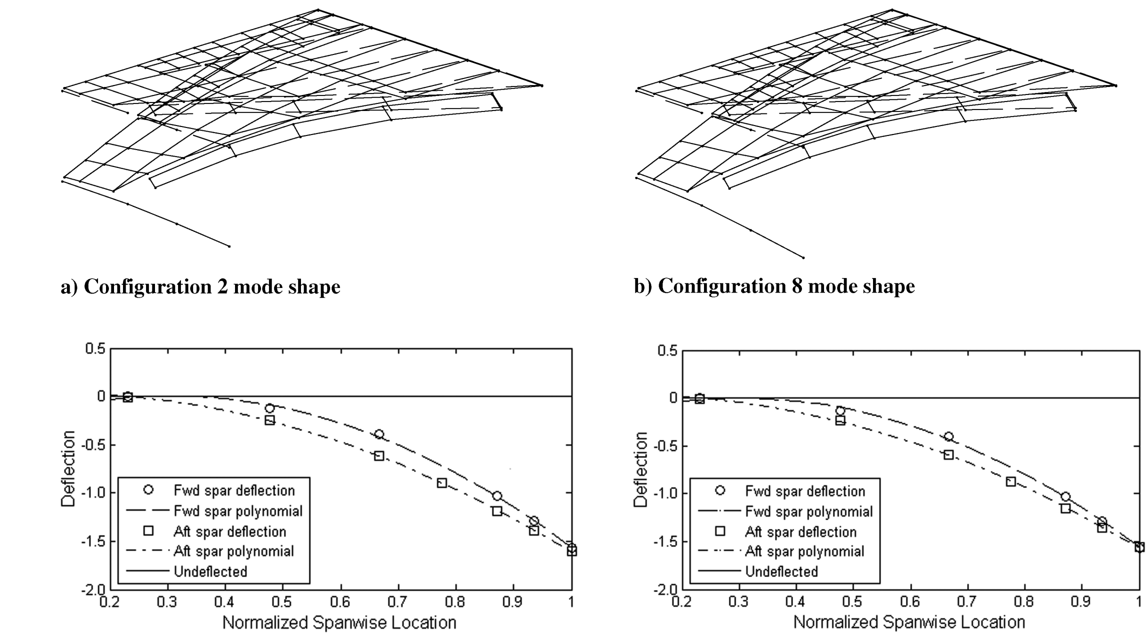


a) Configuration 2 mode shape

b) Configuration 8 mode shape

c) Configuration 2 curve fits

d) Configuration 8 curve fits

Fig. 2 Bending mode shapes and polynomial curve fits.

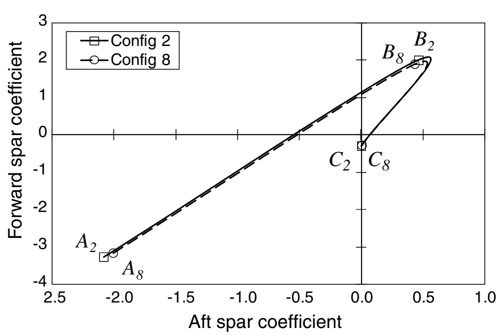
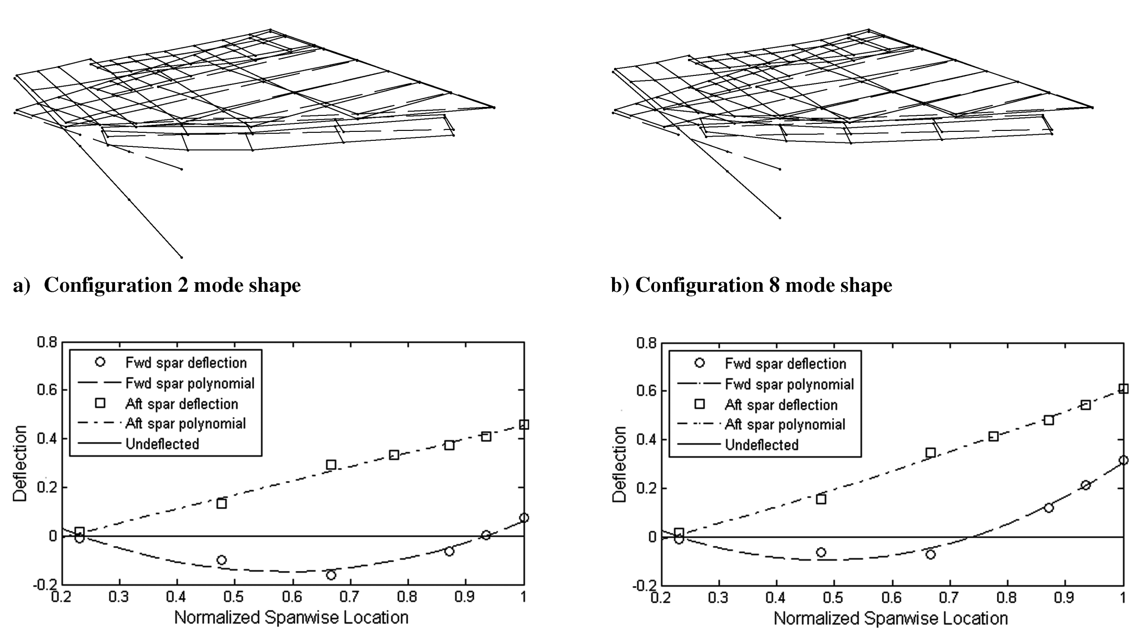


Fig. 3 Bending mode curve-fit coefficient cross plots.

used here to depict the fundamental wing modes (Fig. 1). Selection of these beam members (or, more generally, the leading and trailing edges) allows for a functional reduction of the size of the wing surface mode shape representation while still retaining the significant characteristics of the complete wing modal motion. In essence, the forward and aft spars, when considered together, adequately convey the pertinent bending and torsion deformations of the wing.

Results

The mode shapes for two wing configurations from [8] (weapon configuration numbers 2 and 8) will be examined in this section. The typical flutter mechanism involves 2 to 3 modes of interest, and many times, these modes exhibit bending and torsion characteristics. So,



a) Configuration 2 mode shape

b) Configuration 8 mode shape

c) Configuration 2 curve fits

d) Configuration 8 curve fits

Fig. 4 Torsion mode shapes and polynomial curve fits.

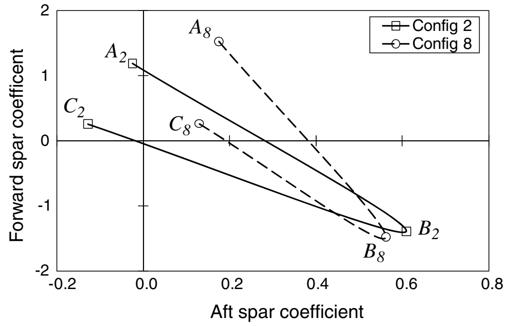


Fig. 5 Torsion mode curve-fit coefficient cross plots.

for each of the wing configurations considered here, the bending and torsion modes, their polynomial fits, and cross plots of their curve-fit coefficients will be presented and compared.

For the wing planform shown in Fig. 1, using the forward and aft spar deflections to describe the modes reduces the required number of deflections from 71 to 13 for each mode of interest. A quadratic polynomial is fit through these deflection points in a least-squares sense using $z = Ay^2 + By + C$ (where y is the spanwise location of the modal deflection; z is the transverse modal deflection; and A, B, and C are the polynomial curve-fit coefficients). This curve-fitting further reduces the size of the wing mode shape quantitative description to 6 coefficients for each mode (3 coefficients each for the forward and aft spars). Additionally, the curve-fit coefficients can be plotted and compared with one another to ascertain similarities between the mode shapes.

Figures 2 and 3 show the results for the wing bending modes of the two wing configurations examined here. Visually, both of these modes look very similar, exhibiting almost pure bending motion with little, if any, wing twist (Figs. 2a and 2b). From the polynomial curve-fit plots (Figs. 2c and 2d) it can be seen that configuration 2 differs slightly from configuration 8 in that it exhibits a slight amount of wing twist at the wingtip. The cross plot (Fig. 3) confirms the similarity of these two bending modes by showing their coefficient curves to be oriented nearly on top of one another.

Figures 4 and 5 show the results for the wing torsion modes. Again, these two modes look very similar from a visual perspective, but a subtle shift in the location of the node line is apparent (Figs. 4a and 4b). The effects of this node line shift are more evident from the polynomial curve-fit plots (Figs. 4c and 4d). Larger deflections are seen at both the forward and aft spar locations for configuration 8, and the fit to the aft spar of configuration 2 is nearly linear. The cross plot (Fig. 5) accentuates these differences even further by showing the coefficient curves of these two modes to have similar shapes but different orientations and extent.

Conclusions

A simple method to quantitatively represent aircraft wing mode shapes was presented in this Note. By functionally reducing the number of deflection points necessary to describe the mode shape, fitting curves through this reduced number of points, then representing the modes only by the curve-fit equation coefficients, the bookkeeping necessary to characterize the modes was significantly reduced. As with many simple methods, there are some inherent strengths and weaknesses.

Mode shapes are typically described by assignment of a subjective name based on the character of the predominant modal motion. Although this method is physical and intuitive, it is sometimes difficult to use to discern nuances among the modes and does not lend itself well to quantitative comparison of modes. The present method overcomes these conventional deficiencies. Here, the number of deformation locations needed to define the modal motion of the wing was effectively reduced by only considering the deflections along the forward and aft spars. Polynomial equations were then fit through the spar deflections to depict the mode shapes in a concise and distinctive manner.

Fitting curves through the deflections at the forward and aft spars is recognized as an approximation of the true mode shape because this approach represents the wing as two separate beams rather than a surface. Because of this simplification, direct physical portrayal of the mode shape is somewhat lost and errors can be introduced through the curve-fitting process. Nonetheless, this approach yields a practical depiction of the general character of the modes and allows for direct comparison of the curve-fit coefficients for each mode.

Comparing the curve-fit coefficients with one another allows one to quickly ascertain similarities and differences between mode shapes. The shape of the cross-plotted coefficient curves (forward spar coefficients vs aft spar coefficients) were seen to accentuate subtle differences between modes and thus make it easier to discern modal nuances.

To retain its concise quantitative advantage, the applicability of this simple method is limited to modes such as free-vibration modes, in which the maximum deflection of every point is reached synchronously during the oscillation cycle. Because of this limitation, the method is somewhat overshadowed by Maxwell's work (as reported in [5]) to depict the wingtip deformation characteristics of the flutter mode shape. In a similar manner, Northington's [6] basis vector approach offers a more mathematically elegant approach that concisely represents wing mode shapes and has the potential to remain concise even when extended to flutter mode shapes.

Many curve-fit possibilities exist, but a quadratic fit was chosen for the work presented here. The choice of a polynomial order that is too high negates the concise advantage that is illustrated in this Note. By representing the fundamental wing modes using only their forward and aft spar deflections, the number of deflection points required to adequately describe the motion of the mode was significantly reduced. This provided an upper bound to the number of numeric descriptors necessary to depict the modes. The lower bound was defined by the quadratic fit, which yielded a nonlinear descriptor of the spar deflections with as few coefficients as possible. It is recognized that more complicated modes may require higher-order fits, but the quadratic fit illustrated the practical bounds on the number of numeric descriptors necessary for implementation of the methodology.

Overall, the method presented here offers practical utility by improving the effectiveness of the study of mode shapes for flutter engineering purposes. Sorting, tracking, screening, and comparing modes for large quantities of analyses is made much more practicable and is accomplished in a simple fashion that allows the user to glean more information from the modes than is typically garnered.

The author is certain that other mode shape comparison methodologies are routinely in use throughout the flutter engineering community, but these works remain presently unpublished. It is hoped that this Note will encourage publication and dissemination of existing in-house methodologies and thereby lead to a greater understanding of the utilization of modes for complex and nonlinear systems.

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